Calculation of Capillary Conductivity from Pressure Plate Outflow Data[1]

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ABSTRACT

The flow equation describing the movement of water out of a soil sample in the pressure plate apparatus has been solved for a special case. It is assumed that during the outflow process (a) the capillary conductivity is approximately constant, and (b) the relation between the water content and the suction is linear. Both of these conditions can be met if the pressure increment which causes the outflow is made sufficiently small. By measuring the outflow from the sample as a function of time and using the solution of the flow equation, the capillary conductivity can be calculated for each suction interval.

Conductivity measurements on three soils indicate that the conductivity decreases approximately as the inverse square to cube of the suction. By using the pressure membrane apparatus, the range of suction over which measurements were made was extended up to 15 bars.

DARCY's law has been used to describe water flow in an unsaturated soil by assuming that the permeability or conductivity is not constant, but rather, is a function of the water content or suction. In vector notation:

$$v = -k(\Theta)\nabla\phi \qquad [1]$$

where v is the volumetric flow velocity, ϕ is the potential function, $k(\Theta)$ is the capillary conductivity, and Θ is the volume of water per unit volume of soil. Combining equation [1] with the equation of continuity:

$$\partial\Theta/\partial t = -\nabla.v \qquad [2]$$

gives

$$\partial\Theta/\partial t = \nabla k(\Theta).\nabla\phi \qquad [3]$$

where t represents time. The nature of the function $k(\Theta)$ is not well known. Gardner (1) concluded from his results that k may be a function of other factors as well as Θ. However, most workers have felt that k is a function of Θ alone. Because of the difficulties involved, relatively few measurements of k have been made, particularly at the lower water contents. The use of tensiometers to measure ϕ has limited the range of suction over which measurements have been made. It is the purpose of this paper to present a method for measuring k by solving equation [3] for the special case of the outflow of water from a soil sample in the pressure plate or pressure membrane apparatus. Experimental data for such a process can be used to calculate k over a range of water content corresponding to a wide range of suction.

Theoretical Analysis

Consider a sample of soil of volume V, cross sectional area A, and height L, which is situated on a porous plate in the pressure plate apparatus (2). Let the initial gage pressure in the chamber be P_i and the water in the soil be in hydraulic equilibrium with water in the buret which is used to receive and measure the outflow. The pressure in the soil water is thus atmospheric. At time t = O, increase the air pressure by an amount ΔP so that the final gage pressure is $P_f = P_i + \Delta P$. This increases the pressure in the soil water by the same ΔP, causing water to flow out of the soil until the pressure in the soil water drops to atmospheric and equilibrium is attained once more. For this flow process, equation [3] becomes in one dimension

$$\frac{\partial\theta}{\partial t} = \frac{1}{\rho g}\frac{\partial}{\partial z}\left(k(\theta)\frac{\partial P}{\partial z}\right) \qquad [4]$$

where, neglecting gravity, ϕ may be taken as the soil water pressure P. In general, this equation is non-linear and difficult to solve analytically. Under certain experimental conditions, assumptions may be made which linearize the equation and make possible its solution. First, we shall assume that ΔP is chosen sufficiently small that k is approximately constant during the flow process. This means that only a small change in the water content will be allowed. This makes it possible to remove k from inside the differential operator. Secondly, we assume that, over this small range of water content, Θ is a linear function of the suction S. While neither of these assumptions is exactly true, any desired degree of approximation may be obtained by selecting ΔP sufficiently small. Noting that, for this case, $P = (P_f - S)$, where P_f is the final gage pressure and S is that soil water suction corresponding to the water content Θ, we can write

$$\Theta(P) = a + bP \qquad [5]$$

where a and b are constants. Substituting [5] in [4] gives

$$\frac{\partial P}{\partial t} = K\frac{\partial^2 P}{\partial z^2} \qquad [6]$$

where K = D/bg. Equation [6] is the same as the heat flow equation with D analogous to the thermal diffusivity and b analogous to the specific heat. With reference to water in soil these may be called the water diffusivity and specific water capacity, respectively.

If one assumes that the pressure in the soil water at the lower boundary is atmospheric at all times, the lower boundary condition is

$$P(O,t) = O \qquad [7a]$$

There is no flow across the upper boundary of the sample. This means that the potential or pressure gradient must vanish at this plane:

$$\left(\frac{\partial P}{\partial z}\right)_{z=L} = 0 \qquad [7b]$$

A convenient method for handling this condition mathematically is to treat the sample as though it had a height of 2L and require that the pressure at this imaginary upper boundary also be atmospheric.

$$P(2L,t) = O \qquad [7c]$$

From symmetry it will be seen that there will be no flow across the plane z = L and condition [7b] is satisfied.

The initial condition requires that

$$P(z,O) = \Delta P \qquad [8]$$

Equations [6], [7], and [8] completely describe the flow problem to the extent that the approximations are valid. Equation [6] may be solved by the method of separation of variables. The values of the arbitrary parameters are selected so that the boundary conditions are satisfied and the resulting solution is expanded in a Fourier sine series to meet the initial condition. The complete solution for P(z,t) is

$$P(z,t) = \frac{4\Delta P}{\pi}\sum_{n=1}^{n=\infty}\frac{1}{n}\sin\left(\frac{n\pi z}{2L}\right)\exp(-\alpha^2 Dt) \qquad [9]$$

(n = 1,3,5,7,...)

where $\alpha = (n\pi/2L)$.

Substituting for $\Theta(z,t)$ from equation [5] gives

$$\theta(z,t) = a + \frac{4b\Delta P}{\pi}\sum_{n=1}^{n=\infty}\frac{1}{n}\sin\left(\frac{n\pi z}{2L}\right)\exp(-\alpha^2 Dt) \qquad [10]$$

(n = 1,3,5,7,...)

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Of more interest is the total water content of the sample which we obtain by integrating Θ from $z = O$ to $z = L$ and multiplying by the area A.

$$W(t) = \int_0^L A\theta(z,t)\,dz = aV + \frac{8b\Delta PV}{\pi^2}\sum_{n=1}^{n=\infty}\frac{1}{n^2}\exp(-\alpha^2 Dt) \quad [11]$$

$(n = 1,3,5,7,\ldots)$

The infinite series converges rapidly and after a few hours only the first term is of any importance. For all $t > 0.3/\alpha^2 D$ the second term is less than 1% of the first.

The initial water content of the soil is, from equation [11],

$$W_i = aV + bV\Delta P$$

and the water content at time $t = \infty$ is

$$W_f = aV$$

The total outflow for the process, Q_o, is just

$$Q_o = (W_i - W_f) = bV\Delta P$$

Solving for b:

$$b = \frac{Q_o}{V\Delta P} \quad [12]$$

Equation [12] allows the calculation of b from the total outflow, sample volume, and pressure increment. The outflow at any time t is $Q(t) = (W_i - W(t))$.

Combining this relationship with equations [11] and [12] yields

$$Q(t) = Q_o\left[1 - \frac{8}{\pi^2}\sum_{n=1}^{n=\infty}\frac{1}{n^2}\exp(-\alpha^2 Dt)\right] \quad [13]$$

$(n = 1,3,5,7,\ldots)$

or, neglecting all but the first term of the series:

$$Q(t) \cong Q_o\left[1 - \frac{8}{\pi^2}\exp(-\alpha^2 Dt)\right] \quad [14]$$

Rearranging and taking the natural logarithms of both sides of the equation gives

$$\ln(Q_o - Q) = \ln\left(\frac{8Q_o}{\pi^2}\right) - \alpha^2 Dt \quad [15]$$

Equation [15] can be used to determine the diffusivity D from experimental data. If the experimentally determined values of $(Q_o - Q)$ are plotted against t on semi-log paper, a straight line with a slope equal to $\alpha^2 D$ and intercept $8Q_o/\pi^2$ should be obtained. If the slope of the line is denoted by B, then $D = B/\alpha^2$ and, since $k = gBb/\alpha^2$, we obtain the result

$$k = \frac{BQ_o\rho g}{V\Delta P\alpha^2} \quad [16]$$

If Q_o is in ml., B in day^{-1}, ΔP in dynes/cm.^{2}, and α^2 in cm.$^{-2}$, k will be in cm./day. Because the second and following terms of the series are neglected, the experimental points will deviate from equation [15] in a region near $t = O$. The capillary conductivity at any suction S may be determined by selecting a pressure interval $P_i < P < P_f$, corresponding to a suction interval $S_i < S < S_f$ remembering that at equilibrium P is numerically equal to S.

Experimental Procedure

As a test of the theory and the validity of the assumptions, the outflow from samples in a pressure membrane and pressure plate chamber was measured and the capillary conductivity calculated by the above method. The experimental procedures and apparatus were essentially the same as those customarily used to make moisture tension versus moisture content measurements. Fragmented soil passing a 2-mm. sieve was packed into a cylinder 20 cm. in diameter and 5 to 10 cm. in height directly on the plate or membrane. The soil was thoroughly wetted and allowed to stand overnight. An initial pressure of from 100 to 150 millibars was applied to the chamber and the sample allowed to come to equilibrium. The outflow was allowed to drip into a buret with the end of the outflow tube level with the center of the soil sample so that the effect of the gravitational field would be a minimum. Data taken during this first pressure increase were not used to calculate k since it was found that at these tensions the boundary condition $P(O,t) = O$ was difficult to satisfy. The resistance to flow across the boundary between the plate and the soil was not negligible compared with the resistance to flow in the soil. This was verified by placing a tensiometer cup at the lower boundary of the soil and observing the pressure at this point by means of a manometer. As the water content decreases, the conductivity of the soil becomes limiting, and it is felt that in most cases when the suction was above 150 to 200 millibars the boundary condition was met satisfactorily. When equilibrium was attained at 200 millibars, the pressure in the chamber was increased by approximately 100 millibars and the outflow recorded as a function of time. When outflow ceased, the pressure was again increased and the process repeated. As the pressure was raised, larger increments of pressure were used, in order to insure removal of sufficient quantities of water for accurate measurement. To obtain pressures above one bar, the pressure membrane apparatus was used and a similar procedure followed.

Results and Discussion

Capillary conductivity measurements were made on three soils: Superstition sand, Pachappa sandy loam, and Chino silty clay loam, having U. S. Salinity Laboratory accession numbers 78, 3045, and 3044, respectively. Two examples of the outflow data for the Pachappa soil are plotted in figure 1. The straight lines were fitted to the experimental points by eye. The expected deviation from the straight line in the vicinity of $t = O$ appears. In some cases there was a tendency for the experimental points to deviate from a straight line near the end of a run. Possibly this was caused by selection of a pressure increment which was too large to satisfy the assumption of a constant k. Such deviations usually occurred at the higher moisture contents where larger quantities of water were removed for each pressure increment. The predicted intercept was not always obtained, but in general the results were very good. There is a possibility that the lower boundary condition was not always met. This would tend to decrease the calculated value of the conductivity but would not necessarily alter the shape of the

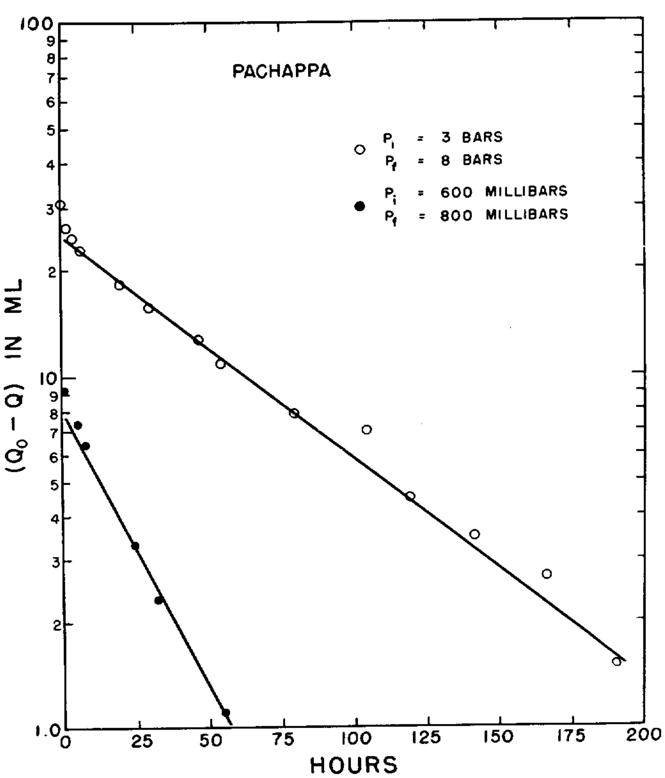

FIG. 1.—Semi-log plot of outflow as a function of time.

outflow curve. The Pachappa and Chino soils were wetted again after their first run and the process repeated on the same sample. In general, the data were more consistent and the results better on the second run. The calculated capillary conductivity values for the three soils are given in table 1 for each initial and final chamber pressure. The data for the Pachappa and Chino soils are for the second run. The data above one bar were taken on a separate sample in the pressure membrane apparatus. The results for the second Pachappa run are plotted in figure 2. The length of the line indicates the pressure interval over which the measurement was made. For purposes of comparison, results obtained by Richards and Moore (3) and Richards *et al.* (4) for the same soil are also plotted. Richards and Moore used a steady state method for measuring the conductivity of fragmented samples while the later results were calculated from suction and water content data taken during a field experiment. The agreement is excellent and indicates that the approximations involved in the derivation are reasonable. Apparently, the structure did not differ sufficiently between the two cases to influence the capillary conductivity to a very great extent. More precise results could be obtained by using a smaller pressure increment. A pressure increment about half of that used here would seem to be preferable.

Table 1.—Capillary conductivity of three soils.

P_i	P_f	Chino	Pachappa	Superstition
bars	bars	k – cm./day		
0.2	0.3	3.0×10^{-3}	11.3×10^{-3}	2.0×10^{-3}
0.3	0.4	1.1×10^{-3}	4.5×10^{-3}	1.7×10^{-3}
0.4	0.5	6.8×10^{-4}	1.7×10^{-3}	1.7×10^{-3}
0.5	0.6	4.4×10^{-4}	8.4×10^{-4}	5.1×10^{-4}
0.6	0.8	3.2×10^{-4}	6.0×10^{-4}	1.7×10^{-4}
0.8	1.0	1.7×10^{-4}	3.4×10^{-4}	1.5×10^{-4}
1.0	2.0	1.4×10^{-4}	1.0×10^{-4}	——
2.0	4.0	0.73×10^{-4}	0.47×10^{-4}	——
4.0	8.0	2.5×10^{-5}	1.4×10^{-5}	——
8.0	14.0	0.84×10^{-5}	0.45×10^{-5}	——
14.0	16.0	0.20×10^{-5}	0.35×10^{-5}	——

From the final water content and the Q_o values for each run, the water content versus water suction curve was constructed. The capillary conductivity of the three soils is plotted as a function of water content in figure 3. While the actual water content range included is small, it represents a major portion of the available water range. It is obvious that the conductivity decreases extremely rapidly with decreasing water content. It may be possible to extend the range of such measurements into the higher water content region by improving the contact between the soil and the porous plate. Calculations based upon the conductivity of the plate show that there is little pressure drop across the plate itself. Increasing the height of the sample would also help to reduce errors due to poor contact.

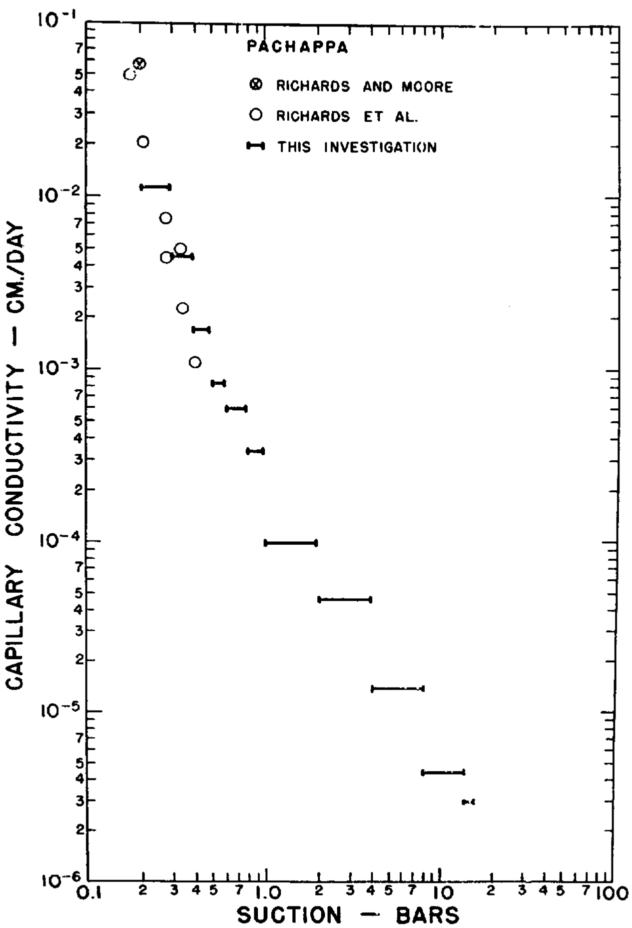

FIG. 2.—Capillary conductivity of Pachappa sandy loam as a function of suction.

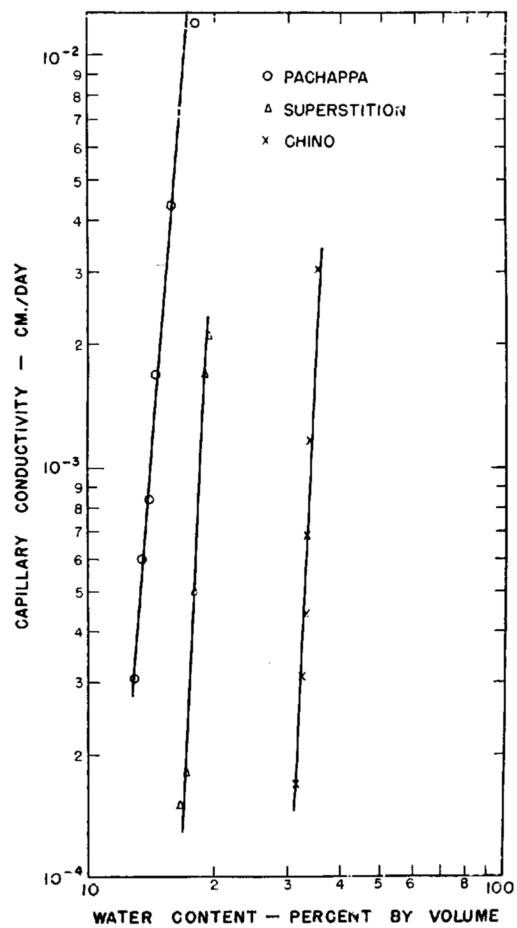

FIG. 3.—Capillary conductivity of three soils as a function of water content.

For analytical purposes, the relations between capillary conductivity and water content or suction can be approximated over a wide range of the variables by functions of the type: $\log k = (\log a' - b' \log S) = (\log c' - d' \log \Theta)$ where a′, b′, c′, and d′ are constants. The nature of these functions yield little hope of linearizing equations [3] and [4] except for a few special cases. However, an analytical expression for $k(\Theta)$ or $k(S)$ will aid considerably in obtaining approximate or numerical solutions of unsaturated flow problems.

The author is indebted to L. A. Richards for suggesting the possibilities of this method and for his advice and encouragement.

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Factors Influencing Swelling and Shrinking in Soils[1]

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ABSTRACT

The mechanism of the swelling process and its influence on soil structure and plant growth is critically evaluated. The tendency of a soil to swell in relation to mineralogical composition, adsorbed organic compounds, exchangeable cations, and iron content is discussed. A description is given of the apparatus developed to measure the swelling pressure of extracted soil clays. A comparison is made of the swelling of extracted soil clays and shrinkage curves obtained from clods taken from four Texas soils in natural structural condition.

It has been assumed generally that a soil which swells is predominantly montmorillonitic, but factors such as organic materials, exchangeable cations, and iron content in addition to its mineralogical composition are important in modifying the tendency of a soil to exhibit swelling characteristics. Removal of iron increased the swelling tendency of extracted clay from some of the soils. Removal of organic matter from the clay increased its swelling capacity, while adsorption of soil conditioners on one of the clays tended to increase its swelling capacity.

Swelling and shrinking of soils influence formation or destruction of favorable structure in the soil. Swelling in some soils helps to cause destruction of aggregates since the tendency is for adjacent aggregates to be forced together so strongly that they rejoin and lose their identity. On the other hand shrinkage favors formation of aggregates from large masses of soil initially in poor structure. Swelling may also be of considerable importance during heavy irrigations or high rainfall when a serious reduction of rate of infiltration may occur in a fine-textured soil which exhibits swelling characteristics. Good soil structure and soil moisture relations are very essential for maximum production of field crops on fine-textured soils so a proper evaluation of the factors governing volume changes in soil should be quite useful.

The primary difficulty in studying factors influencing swelling has been that a suitable method of measurement has not been devised for such studies. The chief method used by most workers in studying the swelling of soil colloids was developed by Winterkorn and Baver (13). In this method the difference in adsorption of water and benzene is used as a measure of swelling. Swelling pressure measurements have been summarized by Marshall (8). It appears that a method based on swelling pressure is more desirable than one based on swelling volumes since the forces involved in formation or destruction of soil structure need to be critically evaluated.

The purpose of this investigation was to develop a simple method for measurement of swelling pressure of extracted soil clays, and evaluate the factors of mineralogical composition, adsorbed organic compounds, exchangeable cations, and iron content in relation to their swelling capacities and consequent influence on soil structure.

Materials and Methods

APPARATUS

A diagram of the apparatus developed to measure swelling pressure of clay is presented in figure 1. The apparatus consists of three principal parts, a brass cylinder-piston-lever arrangement, a laboratory balance, and a solenoid valve controlling water flow into a flask placed on the laboratory balance. A hole, 1 inch in diameter, was bored in the brass cylinder which is the container for the clay. A one gram sample of oven-dried clay which has been screened to pass 20-mesh but retained on a 40-mesh sieve is placed in the cylinder and tamped lightly with the piston to a uniform degree of compaction. The piston fits on top of the clay and contains two holes which allow entry of water to the clay. When the clay swells it creates a pressure on the piston which is amplified by a lever arm with a ratio of 6:1. The lever arm depresses the micro-switch and causes the valve to open in the solenoid valve which permits water to flow by gravity into

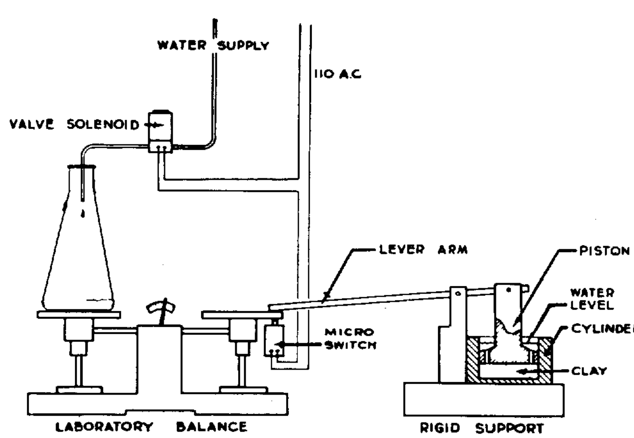

FIG. 1.—Apparatus developed to measure swelling pressure.

[1]Contribution of the Texas Agr. Exp. Sta., College Station, Tex. The work reported in this paper is in partial fulfillment of the requirements for the Ph.D. degree of the Agricultural and Mechanical College of Texas, College Station, Tex. Presented before Div. I. Soil Science Society of America, Davis, Calif., Aug. 16, 1955. Received Aug. 16, 1955.

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